621.3206
I L
Engin
06

TRANSACTIONS
OF THE
ILLUMINATING ENGINEERING SOCIETY

FOUNDED IN LONDON 1909

INCORPORATED 1930

THE LIBRARY OF THE
JUL 23 1956
UNIVERSITY OF ILLINOIS

CONTENTS

Page

Direct Flux, Inter-reflections and Final Luminances in Rectangular Rooms, by R. O. Phillips, B.Arch., A.R.A.I.A., *(Member)* 75

Annual General Meeting - - - 90

Annual Subscriptions - - - - 90

VOLUME 21
No. 4 — 1956

32 VICTORIA ST
LONDON, SW1

Annual Subscription to Non-Members - £2.2.0
Price per Number to Non-Members - 5s. 0d.

XUM

The I.E.S. Council and Committees

Session 1955-1956

Council

President: —A. G. HIGGINS, M.Sc., F.R.I.C. (*Fellow*)

Vice-Presidents:—G. E. L. COMRIE, J. W. HOWELL (*Fellow*), A. G. PENNY (*Fellow*), R. W. STEEL, J. W. STRANGE, Ph.D., A.R.C.Sc. (*Fellow*).

Members of Council:—

Elected Members:—

D. S. ALLOM, C. R. BICKNELL, N. BOYDELL, C. DYKES BROWN, H. CARPENTER, T. CATTEN, J. B. COLLINS, H. F. GILLBE, A. W. GOSTT, H. HEWITT, H. G. JENKINS, A. W. JERVIS, G. E. V. LAMBERT, A. H. OLSON, W. E. RAWSON-BOTTOM, J. REID, W. ROBINSON, N. C. SLATER, C. C. SMITH, J. S. SMYTH, D. A. STRACHAN, W. J. P. WATSON, H. E. G. WATTS, A. H. YOUNG.

Ex-Officio Representatives of Centres:—

J. L. WEST (*Bath and Bristol*), G. R. HANSON (*Birmingham*), R. D. JONES (*Cardiff*), G. E. L. COMRIE (*Edinburgh*), C. STUART (*Glasgow*), A. H. GREEN (*Gloucester and Cheltenham*), R. R. COOPER (*Leeds*), J. L. WALKER (*Leicester*), G. L. BUTLER (*Liverpool*), F. AINSCOW (*Manchester*), J. N. K. RANKIN (*Newcastle*), P. A. MOORE (*Nottingham*), F. A. BENSON (*Sheffield*), J. V. SILVERSTON (*Transvaal*).

Hon. Treasurer:— E. B. SAWYER.

Hon. Secretary:— J. G. HOLMES.

Hon. Editor of Transactions:—C. R. BICKNELL, B.Sc.

All Past-Presidents of the Society are *ex-officio* Members of Council.

Committees

GENERAL PURPOSES COMMITTEE: A. G. Higgins (*President*), E. C. Lennox (*Past-President*), G. E. L. Comrie, J. W. Howell, A. G. Penny, R. W. Steel, J. W. Strange (*Vice-Presidents*), E. B. Sawyer (*Hon. Treasurer*), J. G. Holmes (*Hon. Secretary*), G. R. Hanson (*Birmingham Centre*), W. Berry (*Sheffield Centre*).

PAPERS COMMITTEE: D. S. Allom, T. Catten, H. Carpenter, H. G. Jenkins, G. E. V. Lambert, A. H. Olson, W. Robinson, D. A. Strachan, J. W. Strange, A. H. Young.

EDUCATION COMMITTEE: C. Dykes Brown, H. Hewitt, W. E. Rawson-Bottom, J. Reid, W. J. P. Watson, D. A. Strachan.

CODE COMMITTEE: W. E. Harper, R. G. Hopkinson, A. G. Penny, A. Pott, W. R. Stevens, H. C. Weston.

CENTRES JOINT COMMITTEE: One representative from each Centre and the Vice-Presidents.

AWARDS JUDGING COMMITTEE: J. Reid, N. C. Slater, J. M. Waldram, D. A. Strachan, W. Robinson, C. R. Bicknell.

(The President, Hon. Secretary and Hon. Treasurer are *ex-officio* members of all committees.)

Secretary:—G. F. COLE, 32, Victoria Street, London, S.W.1.

Direct Flux, Inter-reflections, and Final Luminances in Rectangular Rooms

By R. O. PHILLIPS, B.Arch., A.R.A.I.A. *(Member).*

Summary

A method is developed from first principles for finding the average illumination on any surface of a rectangular room due to the luminances of the other surfaces. This method depends on the Hisano "room index," k, and an associated function P_k, derived from the work of Yamauti, which is tabulated for a range of values of k. Illustrative examples show how it may be used to determine the direct flux required to produce a predetermined luminance pattern, or to reverse this procedure. Some comparisons are made between the results obtained by this method and by others, including direct measurement, and the agreement is shown to be generally satisfactory.

(1) Introduction

The report on the "Design of the Visual Field"(1) opened for investigation a whole new range of ideas in lighting design. The stimulating paper by Waldram(2) carried some of these ideas a stage further. In one section of this paper he pointed out how the light released within an interior causes direct illumination of the surfaces, but that in addition inter-reflections build up a gross luminance pattern, i.e., the mean luminance of fairly large parts of the field. He later went on to show how the luminance pattern required could be specified by the concept of "apparent brightness." With the luminance of each surface predetermined, it is then possible to state the required total illumination on it. This may be considered in two parts: (a) The direct illumination; (b) The inter-reflected illumination due to the luminances of all other surfaces visible from it. If (b) can be found, by deducting it from the required total the necessary value of (a) is known, which can then be treated as straightforward illuminating engineering. Waldram suggested that (b) could be found "often sufficiently accurately by inspection."

The later paper by Bellchambers and Ackerman(3) described an investigation into the same problem of finding the direct illumination required to give a predetermined total luminance. The authors pointed out that it is now possible, with the aid of the inter-reflection tables developed by Moon and Spencer, to obtain the final luminance of the room surfaces when the direct illumination is known, but that the reverse procedure is so complex as to be impractical. An attempt was made "to overcome these difficulties from a consideration of fundamental theoretical principles, but a method simple enough for practical application could not be found." The authors therefore turned to an experimental determination of the ratio between total and direct illumination for a range of conditions.

It is evident that design based on the predetermination of the luminance pattern, from which the required direct illumination is derived, could be valuable. However, it seems to require a new approach to the problem of inter-reflections; neither evaluation "by inspection," nor by experiment, necessarily restricted to a limited number of conditions, appears to provide a satisfactory solution. A theoretical treatment, sufficiently simple to be of practical value, should prove preferable. The present paper attempts to provide such a treatment which can be used either to find the illumination required to give a specified luminance pattern, or to reverse this procedure.

The author is on the staff of the New South Wales University of Technology, Sydney, Australia. The manuscript of this paper was first received on August 29, 1955, and in final form October 17, 1955.

(2) Methods of Calculating Illumination

Classical methods of calculation were based on the determination of the direct illumination at a point from a source of known intensity. The Harrison and Anderson "Lumen Method" determined the average illumination of a surface (the working plane) from sources of known flux, as modified by inter-reflection from other surfaces. The Moon and Spencer tables extended the range of the latter to include the determination of the luminance of room surfaces from sources of known flux.

The inter-reflected component of the gross luminance pattern is determined by the average illumination of one surface from others of known luminance. There are well-known methods for finding the illumination at a point from surfaces of given luminance, and the average illumination could thus be found by considering a number of points distributed over the surface, but the procedure would be very laborious.

A more direct method is to find the total flux on one surface from another; if this is divided by the area of the surface, the average illumination is at once determined. The theory necessary for this determination seems first to have been worked out by Lambert[4], and has been further developed by Yamauti[5] and Moon[6]. However, since this work appears to have received little attention it will be briefly developed here in Section 3.1. It will then be shown how, with the aid of the Hisano "equivalent square room," a new function, P_k, may be derived to simplify the work.

(3) Theoretical Development

(3.1) Total flux from one rectangular surface to another

Consider a luminous strip of width a, and of uniform luminance B*, extending vertically to infinity, as shown in Fig. 1. It is required to find the total flux from it to the horizontal surface LMNO, of width a and depth b. Taking O as the origin of co-ordinates on the horizontal plane, let P be the point (x, y). Then E, the illumination at P, is received within the angle OPL, so that:—

$$E = B.\frac{OPL}{2\pi} = B.\frac{OPQ + QPL}{2\pi}$$

$$= \frac{B}{2\pi}\left(\text{arc tan}\frac{x}{y} + \text{arc tan}\frac{a-x}{y}\right)$$

The flux received on an elementary area at P of dimensions $\triangle y$ by $\triangle x$ is $E.\ \triangle y\ \triangle x$. Therefore the total flux, ϕ, on LMNO is given by:—

$$\phi = \frac{B}{2\pi}\int_0^a\int_0^b\left(\text{arc tan}\frac{x}{y} + \text{arc tan}\frac{a-x}{y}\right)\,dy.\,dx$$

It can be shown that the evaluation of this integral gives:—

$$\phi = Ba^2.F\left(\frac{b}{a}\right) \qquad \text{.....(1)}$$

where $F\left(\frac{b}{a}\right)$ denotes a "flux function" of $\frac{b}{a}$.

This function is defined for any variable v by the equation†

---

* In this paper all surfaces are assumed to be uniformly diffusing, and all luminances are expressed in foot-lamberts, or equivalent units. The inclusion in equation (2) of the often-troublesome factor π, then eliminates it from all subsequent calculations.

† By putting ω = arc cot v, the numerator of the above expression may be transformed into the bracketed portion of $\phi_2(\omega)$ given by Yamauti (5) and quoted by Moon (6) and Hisano (7) as:—

$$\phi_2(\omega) = \tfrac{1}{4}\left\{\omega.\cot\omega - \tfrac{1}{2}\ln.\sin.\omega + \tfrac{1}{4}\cot^2\omega.\ln.\cos\omega\right\}$$

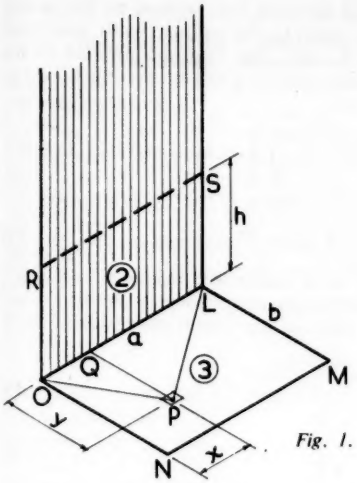


Fig. 1.

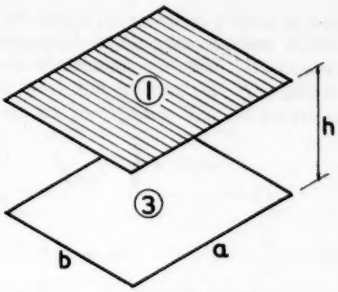


Fig. 2.

$$F(v) = \frac{v \text{ arc cot } v + \frac{1}{4} \ln (1+v^2) - \frac{1}{4} v^2 \ln (1+1/v^2)}{\pi} \quad \ldots\ldots(2)$$

If the luminous strip only extends down to the line RS, at a height h above the horizontal surface, the flux on the latter is given by:—

$$\phi = Ba^2 \left\{ F\left(\frac{\sqrt{b^2+h^2}}{a}\right) - F\left(\frac{h}{a}\right) \right\} \quad \ldots\ldots(3)$$

Denoting surface RSLO as 2, and LMNO as 3, then ϕ_{32}, the flux on 3 from 2, is given by the difference between equations (1) and (3) so that:—

$$\phi_{32} = Ba^2 \left\{ F\left(\frac{b}{a}\right) + F\left(\frac{h}{a}\right) - F\left(\frac{\sqrt{b^2+h^2}}{a}\right) \right\} \quad \ldots\ldots(4)$$

It is to be noted that this expression applies equally well to the flux from LMNO to RSLO. This "reciprocity" of the total flux from one surface to another, for the same uniform luminance, is a perfectly general property, as shown by Moon(6).

If the luminous surface is parallel to the receiving surface as in Fig. 2, ϕ_{31}, the flux from surface 1 to surface 3 is given by:—

$$\phi_{31} = 2B \left[a^2 \left\{ F\left(\frac{\sqrt{b^2+h^2}}{a}\right) - F\left(\frac{h}{a}\right) \right\} + b^2 \left\{ F\left(\frac{\sqrt{a^2+h^2}}{b}\right) - F\left(\frac{h}{b}\right) \right\} \right] \quad \ldots\ldots(5)$$

Methods for dealing with more complex cases are given by Moon. With equations (4) and (5) and a table of values for the "flux function," the total flux received on any surface of a rectangular room from any of the other surfaces can be determined.

(3.2) Flux in the square room

The use of the above equations would still be rather laborious in most cases; fortunately they can be considerably simplified for a square room. Consider a square room of side a and height h and let k equal the ratio between wall and ceiling areas so that:—

$$k = \frac{\text{area of one wall}}{\text{area of ceiling}} = \frac{ah}{a^2} = \frac{h}{a} \quad \ldots\ldots(6)$$

and in such a room let P_k equal the proportion of the total flux emitted by the ceiling which reaches the walls. Let subscripts $_1$, $_2$, $_3$ refer to the ceiling, walls and floor respectively; then the luminance of the ceiling is B_1, etc. The total flux emitted by the ceiling is $B_1 . a^2$, and the flux from ceiling to all four walls is given by putting $b = a$ in equation (4) and multiplying by four. So that:—

$$P_k = \frac{B_1 a^2 . 4 \left[F\left(\frac{a}{a}\right) + F\left(\frac{h}{a}\right) - F\left(\frac{\sqrt{a^2+h^2}}{a}\right) \right]}{B_1 a^2}$$

and as $\frac{h}{a} = k$, then: $P_k = 4\left[F(1) + F(k) - F\left(\sqrt{1+k^2}\right)\right]$* ..... (7)

Hisano[7] showed by "flux functions" that, to a reasonable degree of accuracy, the flux distribution in any rectangular room could be treated as that in the equivalent unit square room of height k. For a rectangular room of horizontal dimensions a by b, and height h, k is given by:—

$$k = \frac{\text{average wall area}}{\text{floor area}} = \frac{1}{4} \frac{\text{total wall area}}{\text{floor area}} \quad \ldots\ldots (8)$$

$$\text{or } k = \frac{h(a+b)}{2ab} \quad \ldots\ldots (8a)$$

Moon and Spencer[8] state that equation (8), the most general expression for k, appears to apply to all rooms with flat ceilings and no re-entrant angles.

(3.3) The unit square room

We are now able to find expressions for the average illuminations on all surfaces of a room in terms of k, P_k, and of B_1, B_2, and B_3, the luminances of ceiling, walls, and floor.

As the ceiling is of unit area, the total flux from it equals B_1. Therefore the flux on the walls is $B_1 . P_k$, and since their area is $4k$, E_{21} the average illumination on the walls due to the ceiling is:—

$$E_{21} = B_1 . \frac{P_k}{4k} \quad \ldots\ldots (9)$$

By symmetry, the flux from the floor to the walls produces an average illumination of:—

$$E_{23} = B_3 \frac{P_k}{4k} \quad \ldots\ldots (10)$$

By the reciprocity principle, the flux from walls to ceiling is equal to the flux from ceiling to walls, for equal luminances. Therefore the average illumination of the ceiling due to walls of luminance B_2 is:—

$$E_{12} = B_2 . P_k \quad \ldots\ldots (11)$$

The average illumination on the working plane at a distance h' below the ceiling, due to inter-reflections, may be treated by considering this plane to be the floor of a unit square room of height k' where:—

$$k' = \frac{h'}{h} k \quad \ldots\ldots (12)$$

By similar methods the illuminations of the other surfaces may be determined; the results are summarised for easy reference in Table 1. As will be shown by

* This equation may be expressed directly in terms of k, as:—

$$P_k = 1 - \frac{1}{\pi}\left[4\sqrt{1+k^2}\ arc\ cot\ \sqrt{1+k^2} - 4k\ arc\ cot\ k + k^2\left\{2ln(1+k^2) - ln\ k^2(2+k^2)\right\}\right].$$

Table 1

Notation and basic relationships between room dimensions, primary flux, and average illuminations and luminances of the room surfaces

Room dimensions	$a \times b$	$k = \dfrac{\text{Area of walls}}{4 \times \text{Area of floor}} = \dfrac{h(a+b)}{2ab}$
Height, floor to ceiling	h	
Height, working plane to ceiling	h'	$k' = \dfrac{h'}{h}.k$

$$P_k = \frac{\text{Flux from unit square ceiling to walls of height } k.}{\text{Total flux from ceiling}}$$

$$= 1 - \frac{1}{\pi}\left[4\sqrt{1+k^2}\ \text{arc cot.}\ \sqrt{1+k^2} - 4k.\ \text{arc cot.}\ k + k^2\left\{2\,ln.\,(1+k^2) - ln.\,k^2\,(2+k^2)\right\}\right]$$

	Ceiling	Walls	Floor
Area (sq. ft.)	$S_1 = a \times b.$	$S_2 = 2h\ (a+b).$	$S_3 = a \times b.$
Reflection factor	ρ_1	ρ_2	ρ_3
Primary (direct) flux (lm.)	ϕ'_1	ϕ'_2	ϕ'_3
Primary illumination (lm./ft.^{2})	$E'_1 = \phi'_1/S_1$	$E'_2 = \phi'_2/S_2$	$E'_3 = \phi'_3/S_3$
Primary luminance (ft.-l.)	$B'_1 = \rho_1\ E'_1$	$B'_2 = \rho_2\ E'_2$	$B'_3 = \rho_3\ E'_3$
Secondary illumination from:			
Ceiling of luminance B_1	—	$E_{21} = B_1.\ P_k/4k$	$E_{31} = B_1.\ (1-P_k)$
Walls of luminance B_2	$E_{12} = B_2.\ P_k$	$E_{22} = B_2\ (1-2P_k/4k)$	$E_{32} = B_2.P_k$
Floor of luminance B_3	$E_{13} = B_3\,(1-P_k)$	$E_{23} = B_3.\ P_k/4k.$	—
Total secondary illumination	$E''_1 = E_{12} + E_{13}$	$E''_2 = E_{21} + E_{22} + E_{23}$	$E''_3 = E_{31} + E_{32}$
Secondary luminance	$B''_1 = \rho_1 E''_1$	$B''_2 = \rho_2 E''_2$	$B''_3 = \rho_3 E''_3$
Illumination on working plane due to ceiling and wall luminances	$E_{w1} = B_1\,(1-P_k').$	$E_{w2} = B_2.\ P_k'$	

examples, they permit the calculation of either the direct illuminations to produce given luminances, or the luminances produced by a known flux.

It is necessary to be able to find the total direct flux (or average illumination) on each major surface. There is still a need for easy methods of achieving these results but Einhorn's "Sector Flux"[9] or Potter and Russell's "Zonal Flux"[10] methods appear to offer possibilities.

Table 2 lists values of P_k for k up to 2, which will rarely be exceeded. The figures have been derived from a table given by Yamauti for his $\phi_2\ (\omega)$, by interpolation with second differences; it is possible that the last figure may err slightly in some cases.

(4) Practical Application

The equations developed in Section 3.2 permit the solution of the inter-reflection problem as suggested by Waldram, but in a more positive manner than by inspection. They also make adjustment between flux, luminance and reflection factors to give the desired result quite straightforward, as shown in the example which follows. The

question of maintenance factors, which are simply corrections applied to the design values for loss in efficiency, is not considered..

Example

Suppose that a doctor's waiting room measuring 12.5 ft. by 12.5 ft. and 12.9 ft. high is to have totally indirect lighting, giving at least 10 lm/ft² on a plane 2.5 ft. above the floor. Taking a value of 0.65 for the average reflection factor of printed paper, the luminance of a magazine page will be about 6.5 ft.-lamberts. To avoid a change of adaptation when looking up at the walls, these will be designed for 6 ft.-lamberts. The floor is to be not less than one-quarter of this, requiring 1.5 ft.-lamberts. For efficiency, the ceiling reflection factor is selected as 0.8, while a restful wall colour tentatively chosen has a reflection factor of 0.4. Since the floor receives only indirect illumination, its reflection factor will be selected after the illumination on it is found.

$$k = \frac{\text{area of walls}}{4 \times \text{area of floor}} = \frac{645}{4 \times 156} = 1.033$$

$$k' = \frac{h'}{h} \cdot k = \frac{12.9 - 2.5}{12.9} \times 1.033 = 0.833$$

From Table 2, $P_k = 0.809$ $\qquad P_k' = 0.749$

$(1 - P_k) = 0.191$ $\qquad (1 - P_k') = 0.251$

$\frac{P_k}{4k} = 0.196$ $\qquad \left(1 - 2\frac{P_k}{4_k}\right) = 0.608$

To provide a minimum of about 10 lm/ft², design will be based on an average of 12 lm/ft². The illumination on the working plane, due to walls of luminance 6 ft.-lamberts, is

$E_{w2} = B_2 \cdot P_k' = 6 \times 0.749 = 4.5$ lm/ft²

$\therefore$ Illumination to be provided by ceiling $= 12 - 4.5 = 7.5$ lm/ft²

$\therefore E_{w1} = B_1 \cdot (1 - P_k') = 7.5$

$$\therefore B_1 = \frac{7.5}{0.251} = 29.8 \text{ ft.-lamberts.}$$

A value of 30 ft.-lamberts will be used. The calculations can now be conveniently set out as below:—

		Ceiling	Walls	Floor
1	Area	156	645	156
2	Reflection factor	0.80	0.40	?
3	Required luminance	30	6	1.5
4	Required final illumination, (3) ÷ (2)	37.5	15	—
5	Illumination due to ceiling luminance	—	5.88	5.72
6	Illumination due to wall luminance	4.85	3.64	4.85
7	Illumination due to floor luminance	0.29	0.29	—
8	Total indirect illumination, (5) + (6) + (7)	5.14	9.81	10.57
9	Required direct illumination, (4)–(8)	32.36	5.19	—
10	Required direct flux (lumens), (1) × (9)	5050	3280	—

As the illumination on the floor is 10.57 lm/ft², and its luminance is to be 1.5 ft.-lamberts, the required reflection factor, ρ_3, is 0.14.

It is known that if four luminaires of a certain type are suspended 2 ft. from

Table 2

Values of the function P_k.

k.	P_k.	diff.	k.	P_k.	diff.	k.	P_k.	diff.	k.	P_k.	diff.
0.01	0.0196		0.51	0.5914		1.01	0.8028		1.51	0.8905	
		189			65			25			11
.02	0385		.52	5979		1.02	8053		1.52	8916	
		184			63			25			11
.03	0569		.53	6042		1.03	8078		1.53	8927	
		179			63			25			11
.04	0748		.54	6105		1.04	8103		1.54	8938	
		174			61			24			11
.05	0922		.55	6166		1.05	8127		1.55	8949	
		169			60			24			11
.06	1091		.56	6226		1.06	8151		1.56	8960	
		165			59			24			11
.07	1256		.57	6285		1.07	8175		1.57	8971	
		162			58			23			11
.08	1418		.58	6343		1.08	8198		1.58	8982	
		158			57			23			10
.09	1576		.59	6400		1.09	8221		1.59	8992	
		154			55			22			10
.10	1730		.60	6455		1.10	8243		1.60	9002	
		151			54			22			10
.11	1881		.61	6509		1.11	8265		1.61	9012	
		147			54			22			10
.12	2028		.62	6563		1.12	8287		1.62	9022	
		144			53			21			10
.13	2172		.63	6616		1.13	8308		1.63	9032	
		141			51			21			10
.14	2313		.64	6667		1.14	8329		1.64	9042	
		138			51			20			10
.15	2451		.65	6718		1.15	8349		1.65	9052	
		135			49			20			9
.16	2586		.66	6767		1.16	8369		1.66	9061	
		132			48			20			9
.17	2718		.67	6815		1.17	8389		1.67	9070	
		129			48			19			9
.18	2847		.68	6863		1.18	8408		1.68	9079	
		127			47			19			9
.19	2974		.69	6910		1.19	8427		1.69	9088	
		124			46			19			9
.20	3098		.70	6956		1.20	8446		1.70	9097	
		121			45			19			9
.21	3219		.71	7001		1.21	8465		1.71	9106	
		119			44			18			8
.22	3338		.72	7045		1.22	8483		1.72	9114	
		116			43			18			9
.23	3454		.73	7088		1.23	8501		1.73	9123	
		114			43			18			8
.24	3568		.74	7131		1.24	8519		1.74	9131	
		112			42			17			8
.25	3680		.75	7173		1.25	8536		1.75	9139	
		109			41			17			8
.26	3789		.76	7214		1.26	8553		1.76	9147	
		107			40			17			8
.27	3896		.77	7254		1.27	8570		1.77	9155	
		105			39			17			8
.28	4001		.78	7293		1.28	8587		1.78	9163	
		103			39			16			8
.29	4104		.79	7332		1.29	8603		1.79	9171	
		101			38			16			8
.30	4205		.80	7370		1.30	8619		1.80	9179	
		98			37			16			7
.31	4303		.81	7407		1.31	8635		1.81	9186	
		97			37			15			8
.32	4400		.82	7444		1.32	8650		1.82	9194	
		95			36			15			7
.33	4495		.83	7480		1.33	8665		1.83	9201	
		93			35			15			7
.34	4588		.84	7515		1.34	8680		1.84	9208	
		91			35			15			7
.35	4679		.85	7550		1.35	8695		1.85	9215	
		89			34			15			8
.36	4768		.86	7584		1.36	8710		1.86	9223	
		87			33			15			7
.37	4855		.87	7617		1.37	8725		1.87	9230	
		86			33			14			7
.38	4941		.88	7650		1.38	8739		1.88	9237	
		84			32			14			7
.39	5026		.89	7682		1.39	8753		1.89	9244	
		82			32			14			7
.40	5108		.90	7714		1.40	8767		1.90	9251	
		80			31			13			6
.41	5188		.91	7745		1.41	8780		1.91	9257	
		79			31			13			7
.42	5267		.92	7776		1.42	8793		1.92	9264	
		78			30			13			7
.43	5345		.93	7806		1.43	8806		1.93	9271	
		76			29			13			6
.44	5421		.94	7835		1.44	8819		1.94	9277	
		75			29			13			7
.45	5496		.95	7864		1.45	8832		1.95	9284	
		73			29			12			6
.46	5569		.96	7893		1.46	8844		1.96	9290	
		72			28			13			6
.47	5641		.97	7921		1.47	8857		1.97	9296	
		70			27			12			6
.48	5711		.98	7948		1.48	8869		1.98	9302	
		69			27			12			6
.49	5780		.99	7975		1.49	8881		1.99	9308	
		68			27			12			6
.50	5848		1.00	8002		1.50	8893		2.00	9314	
		66			26			12			

the ceiling, 0.746 of their total effective output will reach the ceiling. Therefore:—

$$\text{Required output} = \frac{5{,}050}{0.746} = 6{,}770 \text{ lumens.}$$

Of this total, the remaining 0.254 will reach the walls, so that:—

$$\text{Direct flux to walls} = 6{,}770 \times 0.254 = 1{,}720 \text{ lumens.}$$

This is roughly only one-half the required value. However, the required luminance of 6 ft.-lamberts on the walls can be maintained by adjusting their reflection factor. If this is done, we will still have the same indirect illumination of 9.81 lm/ft². The average direct illumination will now be:—

$$\frac{1{,}720}{645} = 2.66 \text{ lm/ft}^2$$

giving a total of 12.47 lm/ft². To obtain a luminance of 6, we require a reflection factor for ρ_2 of 0.48.

A value of 0.5 will be adopted, which should result in a luminance of 6.24 ft.-lamberts. The final average illumination on the working plane will then be:—

$$\text{From the ceiling, } E_{w1} = 30 \times 0.251 = 7.53$$
$$\text{From the walls, } E_{w2} = 6.24 \times 0.749 = 4.67$$
$$\text{Total, } E_{av} = 12.20 \text{ lm/ft}^2.$$

This completes the design procedure. It can be seen that it is quite flexible, and that adjustments may be made without re-calculating from the beginning.

(5) Inter-reflections from Primary Flux

The equations of Table 1 can also aid in finding the illumination and luminances which result from a known primary flux. However, several methods are possible and some additional factors must be considered, as discussed below.

(5.1) Method based on Table 1

The data of Table 1 would permit us to trace reflected flux through stage after stage of the inter-reflection process. For example, with a direct illumination E'_1, on the ceiling of reflection factor ρ_1, its luminance $B'_1 = \rho_1 E'_1$.

Then the luminance B_{31}, of the floor, due to the ceiling of luminance B'_1, is:—

$$B_{31} = \rho_1 \rho_3 E'_1 (1 - P_k)$$

This value of B_{31} will in turn produce a luminance on the walls of:—

$$B_{23} = \rho_1 c_2 \rho_3 E'_1 (1 - P_k) \frac{P_k}{4k} \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

It is evident that this process could be continued indefinitely for each surface in turn, but that it would rapidly become quite unmanageable.

(5.2) Summation by integrating sphere theory

Alternatively we can turn to the theory of the integrating sphere, as has been done by Dresler[11], to determine the reflected component in daylight. Suppose that flux ϕ' is released in an enclosure of surface area S, and uniform reflection factor ρ. After one reflection the added flux will be $\phi'\rho$, after two reflections $\phi'\rho^2$, and so on. Therefore the average illumination, E, after an infinite number of reflections will be found by adding these terms and dividing throughout by S, giving:—

$$E = \frac{\phi'}{S} + \frac{\phi'\rho}{S} + \frac{\phi'\rho^2}{S} + \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

This is a geometric series, and since ρ is by nature positive and less than unity, the

sum of all terms after and including any given term is found by dividing that term by $1-\rho$, so that:—

$$E = \frac{\phi'}{S(1-\rho)} \qquad \text{(15a)}$$

$$\text{or } E = \frac{\phi'}{S} + \frac{\phi'\rho}{S(1-\rho)} \qquad \text{(15b)}$$

$$\text{or } E = \frac{\phi'}{S} + \frac{\phi'\rho}{S} + \frac{\phi'\rho^2}{S(1-\rho)} \qquad \text{(15c)}$$

To apply these equations to a room where the various surfaces have different reflection factors, it is necessary to substitute for ρ a value $\bar{\rho}$, the mean reflection factor.

Equation (13) shows that primary flux reflected in turn from the ceiling and the floor produces on the walls a luminance given by an expression containing the product $\rho_1\rho_2\rho_3$. We can substitute for this $(\bar{\rho})^3$ where $\bar{\rho}$ is the mean reflection factor at each reflection. It is evident that in this case $\bar{\rho}$ is the cube root of $\rho_1\rho_2\rho_3$, i.e., it is the geometric mean of these quantities.

It seems reasonable to assume that, after many reflections, the number of times any small element of flux has been reflected from each of the surfaces will be proportional to the relative areas of ceiling, walls, and floor, i.e., to 1, $4k$, and 1 respectively. Then the weighted geometric mean is given by:—

$$\log \bar{\rho} = \frac{\log \rho_1 + 4k.\log \rho_2 + \log \rho_3}{2 + 4k} \qquad \text{(16)}$$

Substituting the value of $\bar{\rho}$ so found for ρ in equation (15a) would therefore give at least a first approximation to the average illumination of all surfaces in the room. However, where the reflection factors of the various surfaces differ by ordinary amounts this method is too inexact.

Improved results can be obtained by applying the summation process to the second term, as in equation (15b). It is convenient to adopt the convention of adding (′) to all photometric quantities due to primary flux, and (″) to those due to secondary (i.e., once reflected) flux. The term ϕ'/S, which represents the primary illumination, is then treated separately for each surface, giving:—

$$\text{for the ceiling, } E'_1 = \phi'_1/S_1 \qquad \text{(17a)}$$

$$\text{for the walls, } E'_2 = \phi'_2/S_2 \qquad \text{(17b)}$$

$$\text{for the floor, } E'_3 = \phi'_3/S_3 \qquad \text{(17c)}$$

The numerator of the second term of equation (15b), $\phi'\rho$, gives the flux which has been reflected once in a room of uniform reflection factor. In the non-uniform room this is obtained by adding the first-reflected fluxes from each surface. Hence the average added illumination due to flux which has been reflected once or more is obtained by adding the separate reflected fluxes and dividing as in equation (15b), by $(1-\bar{\rho})$ and by the total surface area.

The final average illumination on any surface is then found by adding this value to the primary illumination on it.

If this method is applied to the example of Section 4, adopting the values of

primary flux and reflection factors determined in the design procedure, a comparison of the results with the designed values is shown below:—

		Design value	*Diff.*
Final luminance of ceiling (ft.-L.)	33.2	30	+10.7%
„ „ „ walls „	5.9	6.24	−5.6%
„ „ „ floor „	1.27	1.5	−15.3%
Illumination on working plane (lm/ft²)	12.45	12.20	+2.0%

The agreement is reasonably good, but in a case such as this, where most of the reflected flux comes from one surface (here the ceiling), and hence would mainly illuminate the *other* surfaces, the summation process spreads it evenly over *all* surfaces. This results in values too high for the brightest surface, and too low for the others. In many cases errors of this order might be unimportant, but accuracy can be improved by delaying the summation process, as shown below.

(5.3) Combined use of Table 1 and summation

In this case we consider:

(a) The direct illumination on each surface.

(b) The indirect illumination on each surface due to first reflection from each other surface, by Table 1.

(c) The later inter-reflected illumination.

These are equivalent to the three terms of equation (15c).

The average primary illumination on each surface is first found, and when multiplied by its reflection factor, gives its primary luminance.

The secondary illumination of each surface due to the primary luminance of each other surface can then be found from Table 1. These secondary illuminations are added for each surface, and the result multiplied by the reflection factor, giving the secondary luminances B''_1, B''_2 and B''_3 of ceiling, walls and floor. If these values are multiplied by their areas and added, we obtain the total flux after *two* reflections. Then $\bar{E}''$, the average inter-reflected illumination due to flux which has been reflected two or more times, is found by dividing the result by the total surface area and by $(1 - \bar{\rho})$, found from equation (16), giving:—

$$\bar{E}'' = \frac{B''_1 + 4k \,.\, B''_2 + B''_3}{(2 + 4k)(1 - \bar{\rho})} \qquad \text{..... (18)}$$

For each surface, the value of E'' is multiplied by the appropriate reflection factor, giving the added luminance due to all later inter-reflections. The three values of luminance for each surface (primary, secondary, and inter-reflected) are then added to give the final luminance. The average indirect illumination on the working plane is found from the final luminances of the ceiling and the walls, any direct illumination being simply added to the indirect values.

Example

As an example of this method, the values of primary flux and reflection factors selected in Section 4 will be used, as a check, in the reverse procedure.

BASIC DATA

$k = 1.033$	$k' = 0.833$
$P_k = 0.809$	$P_k' = 0.749$
$(1-P_k) = 0.191$	$(1-P_k') = 0.251$
$P_k/4k = 0.196$	$\bar{\rho} = 0.438$ (from eqn. 16)
$(1-2P_k/4k) = 0.608$	$(1-\bar{\rho}) = 0.562$

CALCULATIONS

		Ceiling	Walls	Floor
1	Primary flux	5050	1720	—
2	Area	156	645	156
3	Reflection factor	0.80	0.50	0.14
4	Primary illumination, (1) ÷ (2)	32.4	2.67	—
5	Primary luminance, (3) × (4)	25.92	1.34	—
6	Secondary illumination due to ceiling	—	5.08	4.95
7	Secondary illumination due to walls	1.08	0.81	1.08
8	Total secondary illumination, (6) + (7)	1.08	5.89	6.03
9	Secondary luminance, (3) × (8)	0.86	2.95	0.85
10	Luminance due to $\bar{E}''$ (below), (3) × $\bar{E}''$	3.22	2.02	0.57
11	Final luminance, (5) + (9) + (10)	30.00	6.31	1.42
	Designed Value	30	6.24	1.5
	Difference from designed value	0	+ 1.1 %	− 5.3 %

$$\bar{E}'' = \frac{B''_1 + 4k.B''_2 + B''_3}{(2 + 4k)\ (1-\bar{\rho})} = \frac{0.86 + (4.132 \times 2.95) + 0.85}{6.132 \times 0.562}$$

$$= 4.04 \text{ lm/ft}^2$$

Illumination on working plane due to ceiling $= B_1(1-P_k') = 30 \times 0.251 =$ 7.53
Illumination on working plane due to walls $= B_2 P_k' = 6.31 \times 0.749 =$ 4.72
Total $= 12.25 \text{ lm/ft}^2$
Design Value $= 12.20 \text{ lm/ft}^2$

The agreement between the pre-determined design values and those calculated by this method is about as close as could reasonably be expected. Therefore the method gives consistent results whether proceeding from luminance pattern to required primary flux distribution, or vice versa. However, this does not demonstrate whether the results achieved would agree with those obtained by measurement or by other methods of calculation. This question will be examined in Section 6.

(5·4) Variation to allow for grading of wall luminance

These methods all give *average* values for the luminances of the various surfaces; in most cases this will possibly be adequate. However, with lighting systems which are mainly indirect, there will be a definite grading of wall luminance downwards from top to bottom. This can quite easily be found by dividing the height of the walls into n horizontal strips. If the distance from the ceiling to the bottom of the first strip is $k1$, to the second $k2$, etc., then the flux from the ceiling to the *first* strip is $B_1\ P_{k1}$ and to the *first plus the second* is $B_1\ P_{k2}$, etc. Therefore the flux from the ceiling to the second strip alone is $B_1\ (P_{k2} - P_{k1})$ and so on. The illumination on each is found by dividing the above values by $4k/n$, the area of each strip. The initial procedure is as before, until the average final luminance of all surfaces is found. The illumination on one wall is then approximately the average value due to the other walls, plus values due to the ceiling found as above, plus illumination due to the floor. The latter may be treated as an overall average, or in strips as for the ceiling; unless the floor luminance is high it will usually make little difference which method is used.

Example

To illustrate this method, a somewhat improbable arrangement of a high room, not square, with a luminous ceiling has been chosen. This provides a severe test of the method and can also be compared with measurement, as shown in Section 6.

A room 13 ft. by 20 ft. on plan, and 20 ft. high, is lighted by a translucent glass ceiling, illuminated from above so as to give a primary luminance, B'_1, of 100 ft.-lamberts. The reflection factors of the glass ceiling, the walls, and the floor are 0.70, 0.58, and 0.20, and $k = 1.27$.

The average final luminances of the surfaces are found, exactly as in the example of Section 5.3, to be:—

Ceiling ..... 110.4 ft.-lamberts.
Walls ..... 18.4 „ „
Floor ..... 5.84 „ „

The graded illumination due to the ceiling is then found as described above. Consider the walls as divided into five horizontal strips, each 4 ft. deep. In terms of k, each strip is $k/5$ deep, i.e., 0.254, and the area of each strip is $4k/5$, i.e., 1.016. The work can be set out as below.

Depth below ceiling (ft.) (1)	Equiv. k (2)	Value of P_k (3)	Difference (P_k for strip) (4)	Illumination due to ceiling (4) $\times \frac{110.4}{1.016}$	Final illumination of strip (lm/ft²)
0	0.0	0.0	0.3723	40.5	53.7
4	0.254	0.3723	0.2178	23.7	36.9
8	0.508	0.5901	0.1321	14.4	27.6
12	0.762	0.7222	0.0821	8.9	22.1
16	1.016	0.8043	0.0526	5.7	18.9
20	1.27	0.8569			

Column (4) represents the proportion of the total flux from the ceiling reaching the strip in question. The resultant illumination is found by multiplying by the final ceiling luminance (110.4) and dividing by the area of the strip. The values in the last column are found by adding the illumination due to the final luminances of the other walls and the floor, 13.2 lm/ft². The final luminances would, of course, be obtained by multiplying by the reflection factor; the results have been left in the form shown for later comparison with measured values.

(6) Accuracy of the Results

It seems probable that the large number of variables involved in the general solution of the inter-reflection problem would make any completely rigorous approach either impossible or so complex as to be quite impractical. Certain compromises and approximations therefore appear inevitable; the best that can be hoped for is that they will not lead to errors large enough to be important in practice.

The major approximation adopted in the method here described is that of assuming that all illuminations may be averaged over the surfaces on which they are

incident (except in the method of 5.4). An attempt will be made to assess the errors involved by comparison with other data.

The original Harrison and Anderson data were frankly empirical. Recently Potter and Russell[10] have re-measured "Utilisation Factors"* using more refined techniques, comparing the measured results with those obtained by the Moon and Spencer "Interflectance"* tables. They demonstrated that, in general, the measured values are higher than those calculated. In the discussion following this paper, Dr. Spencer pointed out that the integral equations on which the tables are based required, for their solution, the use of "kernels" in which exponential approximations were substituted for the true values.

For example in a room with perfectly diffusing luminous ceiling and black walls and floor, the "interflectance," f_2 (the ratio between flux to working plane, and flux from ceiling), is taken as:—

$f_2 = e^{-2k'}$ (with the notation used herein)

Dr. Spencer goes on to point out that exact equations can be derived for specific room shapes, and gives for a square room an expression which is equivalent to $(1-P_k')$ of this paper, as of course it should be. Another compromise adopted in the Moon and Spencer tables is to standardise on a working plane at one-quarter of the room height. so that k' is always $0.75k$. The degree of error involved in this approximation can be seen below:—

k	0.0	0.2	0.4	0.6	0.8	1.0	1.2
k'	0.0	0.15	0.30	0.45	0.60	0.75	0.90
$(1-P_k')$	1.0000	.7550	.5795	.4504	.3545	.2827	.2286
$e^{-2k'}$	1.0000	.7408	.5488	.4066	.3012	.2231	.1653
error	0	−1.9 p.c.	−5.3 p.c.	−9.7 p.c.	−15 p.c.	−21 p.c.	−27.5 p.c.

The example of Section 5.1 was deliberately chosen so as to fit exactly one set of the Potter and Russell measurements (namely that of the second column of figures in Table V). The measurements extended only to the determination of the "Utilance." In the example of Section 5.3, this is:—

$$\frac{E_{av} \times \text{Floor Area}}{\text{Total Flux}} = \frac{12.25 \times 156}{6{,}770} = 0.282$$

This differs from the measured value of 0.300 by −6 per cent. The value calculated from the Moon and Spencer equations for the exact room dimensions and the true height of working plane is 0.259, a difference of −13.6 per cent. Comparison with others of the Potter and Russell data gives similar results as shown below:—

Potter and Russell Table and Column No.	V,1	V,4	V,6	IV,7
Type of luminaire	Indirect	Indirect	Indirect	Direct
k	1.20	0.825	0.70	0.53
Utilance, "interflectance" method	0.210	0.339	0.411	0.849
Utilance, method of Section 5.3	0.242	0.349	0.423	0.850
Utilance, measured	0.251	0.378	0.438	0.885

Unfortunately no measurements of the surface luminances of the experimental room are available. A comparison has therefore been made with the normal Moon and Spencer tables, using the tabulated data nearest to those of the actual room of Section 5.3. The results are as shown at the top of the next page. Agreement is surprisingly close.

A word should be said about the determination of $\bar{\rho}$ as the weighted geometric mean, in place of the weighted arithmetic mean, as used by other workers. The

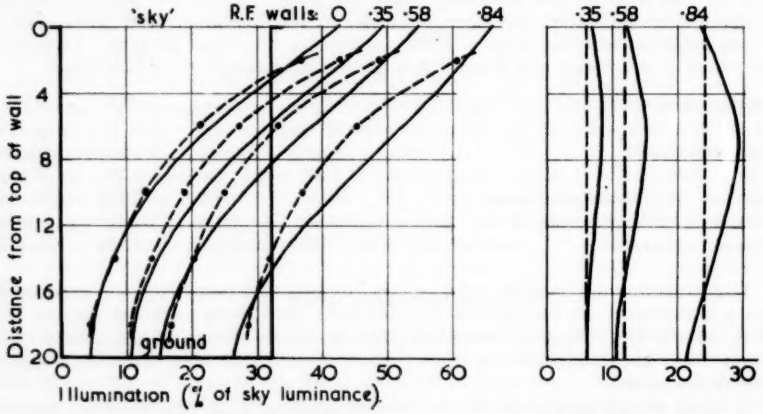
* Both these terms are equivalent to what the C.I.E. has now recommended should be called "Utilance," i.e., the ratio between flux to working plane and flux from luminaire (as distinct from flux from lamps).

	Average Illumination lm/ft²	Luminances of Ceiling ft.-l.	Walls ft.-l.	Floor ft.-l.
Due to 5050 lm. on ceiling	10.3	30.0	4.25	1.01
Due to 1720 lm. on walls	2.0	1.5	2.15	0.28
Totals	12.3	31.5	6.40	1.29
Values from Section 5.3	12.25	30.0	6.31	1.42

theoretical justification for this has been given. Some comparisons have also been made between published tables and calculations of the type shown above, and better agreement has been obtained with the geometric mean.

As one part of his extensive experimental investigation of daylight, Pleijel[12] measured the illumination on the walls of courtyards. A model court was used in which the reflection factors of the surfaces could be changed, the "sky" consisting of a sheet of flashed opal glass, illuminated from above and resting on the tops of the walls. This is precisely equivalent to a room lighted by a luminous ceiling, and one set of measurements has been chosen as the example of Section 5.4.

A series of measurements was taken of the illumination at points on the vertical centre line of one long wall, first with all surfaces matt black, and then with the reflection factors of the ground 0.2, and of the walls 0.2, 0.35, 0.58, and 0.84. Calculations have been shown for the value of 0.58 for the walls; illuminations have also been calculated for values of 0.35 and 0.84. The results are shown in Fig. 3a. Since Pleijel expressed his measurements as daylight factors, i.e., as percentages of the illumination from a complete hemisphere of sky with the luminance of the actual sky, the calculated data have been plotted in the same form. The full-line curves are measured; the broken curves were formed by plotting at the centre of each strip of wall the calculated average illumination of the strip and joining the points so found. It can be seen that the agreement is reasonable, but that the shapes of the calculated



(a) Total illumination *(b) Reflected component only*

Fig. 3. Illuminations on the walls of a courtyard 20m × 13m × 20m high. The full lines are Plejel's measured Vilnes, the broken lines were calculated.

curves are incorrect. In Fig. 3b, the results are replotted showing only the reflected component of the total illumination. This indicates the degree of error introduced by considering only the *average* illumination contributed by walls and floor.

(7) Conclusion

A method has been developed from first principles by which the direct flux, the inter-reflected flux, and the resultant illuminations and luminances on room surfaces can be related. This method pre-supposes that the direct flux to each surface can be determined; the data here required may not as yet always be available in readily usable form, but work on this aspect of design is proceeding in several countries. However, when this information is available, the remaining calculations require nothing more elaborate than the table of values of P_k presented here and a slide-rule, as used for the illustrative examples.

Several examples have shown that the method appears to give values which are self-consistent, whether working from final luminances to primary flux or vice versa. Comparison with values obtained by measurement and the Moon and Spencer "interflectance" tables shows generally satisfactory agreement. Many more such comparisons would be required before it would be possible to say that one method of calculation was more precise than the other. When the designer has the information he requires in the form of published tables there is thus no reason to go beyond them.

However, the methods here presented do appear to have the following advantages:—

(1) They are more suitable for use in *design*, that is in proceeding from the luminance pattern to the required flux distribution, as illustrated in Section 4.

(2) For dealing with special cases, not covered by the usual tables, they are easier to apply. The only complex mathematical expression required is that for P_k. This is related purely to the geometry of the room, and can be tabulated once and for all, as none of the other variables enter into it. The "interflectance" method also contains complex equations into which, however, all three values of surface reflection factors enter, so that a change of one of them requires recalculation almost from the beginning.

It is the author's hope that other workers will test the methods of calculation described here against experimental data, to secure further evidence as to whether the calculated and observed results agree as well as they appear to do in the admittedly limited number of cases here presented.

References

(1) "The Design of the Visual Field." Report of Sub-Committee of the National Illumination Committee of Great Britain. Trans. Illum. Eng. Soc. (London), **18**, 225 (1953).

(2) Waldram, J. M.: "Studies in Interior Lighting." Trans. Illum. Eng. Soc. (London), **19**, 95 (1954).

(3) Bellchambers, H. E., and Ackerman, K. R.: "Lighting Design and Brightness Patterns—a Preliminary Investigation." Trans. Illum. Eng. Soc. (London), **20**, 79 (1955).

(4) Lambert: "Photometrie," Ostwalds Klassiker der Exakten Wissenschaften, pp. 67-73. (Reference given by Yamauti.)

(5) Yamauti, Z.: "The amount of Flux Incident to Rectangular Floor Through Rectangular Windows," Researches Electrotechnical Lab., Tokyo, No. 250 (1929).

(6) Moon, Parry: "Scientific Basis of Illuminating Engineering," pp. 332-344. (McGraw-Hill, 1936).

(7) Hisano, K.: "Light flux distribution in a rectangular parallelepiped and its simplying scale." English Translation, Illum. Eng. (N.Y.), **41**, 232 (1946).

(8) Moon, Parry and Spencer, D.E.: "Lighting Design" (Addison-Wesley Press, 1947).

(9) Einhorn, H. D.: Trans. Illum. Eng. Soc. (London), **16**, 101 (1951).

(10) Potter, W. M., and Russell, A. H.: "Measured Utilisation Factors," Illum. Eng. (N.Y.), **49**, 137 (1954).

(11) Dresler, A.: "The Reflected Component in Daylighting Design." Trans. Illum. Eng Soc. (London), **19**, 50 (1954).

(12) Pleijel, G.: "Daylight Investigation," Report No. 17 of the State Committee for Building Research, Stockholm (1949).

Annual General Meeting

The annual general meeting of the Illuminating Engineering Society was held at the Royal Hall, Harrogate, at 2.30 p.m. on Friday, May 11, 1956. The president, Mr. A. G. Higgins, was in the chair.

In presenting the report of the council and the accounts for the year ending December 31, 1955, the president drew attention to the increase in membership of 84 during the year. He emphasised the need for new members in all classes and said that there were many organisations in the industry who should be encouraged to become sustaining members. The president also referred to the activities of the centres during the year and to the new edition of the I.E.S. Code. The hon. treasurer explained that the excess of income over expenditure during the year was due to fortuitous circumstances and he drew attention to rising costs.

The adoption of the report and accounts was proposed by Mr. J. S. McCulloch, who also proposed a vote of thanks to the president, officers and members of council for their conduct of the society's affairs during the year. This was seconded by Mr. A. G. Smith. The report and accounts were declared approved.

The president also said that the council had given considerable thought to a revised constitution for the society and it was hoped to bring this to the attention of members in the near future.

Dealing with the election of officers and members of council for the 1956-57 session the president said that no other nominations having been received the following were declared elected :—

President: Dr. W. E. Harper.

Vice-Presidents: Dr. R. G. Hopkinson, Mr. J. S. McCulloch, Mr. E. B. Sawyer, Mr. C. C. Smith, Mr. W. T. F. Souter.

Hon. Treasurer: Mr. W. Robinson.

Hon. Secretary: Mr. J. G. Holmes.

Hon. Editor of Transactions: Mr. C. R. Bicknell.

Members of Council: Mr. A. D. S. Atkinson, Dr. H. H. Ballin, Mr. D. E. Beard, Mr. H. G. Campbell, Mr. J. B. Harris, Mr. A. Pott, Mr. G. C. Small, Mr. J. F. Stanley.

After the remuneration of the auditors for the current year had been agreed the meeting was declared closed.

Annual Subscriptions

At an extraordinary general meeting of the society, held at the Royal Hall, Harrogate, at 2.45 p.m. on Friday, May 11, 1956, resolutions amending the by-laws in respect of annual subscriptions and the class of overseas member were considered and approved.

As from December 31, 1956, the class of overseas member will be discontinued and the following new rates of annual subscription will come into effect :—

	£	s.
Fellows	4	10
Corporate Members (except Fellows)	3	10
Students (ages 16-24)	1	0
Sustaining Members (minimum)	7	7

956-57
other
d the
—

inson,
E. B.
. T. F.

n.
s.

cknell.
D. S.
D. E.
. J. B.
Small,

the
been
closed.

S

eeting
Hall,
riday,
ng the
scrip-
ember

e class
iscon-
tes of
into

£ s.
4 10

3 10
1 0
7 7

London).

XUM

Printed by
ARGUS PRESS, LTD.,
Temple-avenue and
Tudor - street,
London, E.C.4,
England.